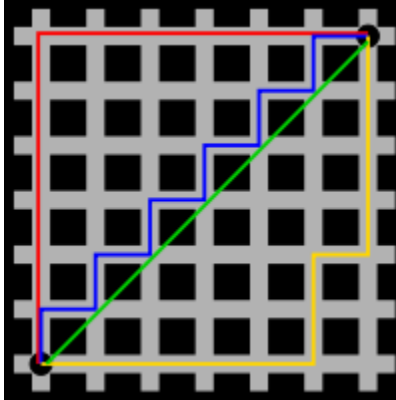# The Manhattan Metric

by Miles Mathis

First posted December 11, 2012

Taxicab Geometry. I recently posted an update to [my controversial π=4 paper,](my controversial π=4 paper) showing that my analysis is basically equivalent to the Manhattan metric of Hilbert, also known as Taxicab Geometry. Somehow it took me four years to figure that out. Actually, I didn't *know* until a few days ago that π=4 in the Manhattan metric. I knew the Manhattan metric only as a grid metric, and had never seen it applied to a circle. I had to be sent to the Wikipedia page on the Manhattan metric, where it says that π=4. Only then was I able to make the connection.

You see, I had come to π=4 from a completely different direction, one that had nothing to do with Hilbert or the Manhattan metric. Hilbert was working on the distance between given points in a field, while I was studying orbits. Hilbert was playing with formalisms, as usual, while I was rigorously analyzing circle kinematics. Hilbert was looking at linear distances, while I was measuring curves. Specifically, I was trying to clear up longstanding disclarities in the classical orbital math, especially the equation v = 2πr/t. Since the distance 2πr is a circle, it must be a curve rather than a straight line. But, by definition, you can't express a velocity as a curve over a time. A velocity is a vector, and hence is always linear. A curve is multiple motions or vectors during the same time, so a curve must always be an acceleration. In unwinding this problem, I was finally forced to redo the orbital math from the ground up, which brought me at last to my problem with π.

In recently tying my paper to Hilbert's metric, I thought the veils would fall for everyone just as they had for me. I thought all I had to do was add a couple of paragraphs to my original paper, and everyone in the world would see what I saw. But, as usual, it doesn't seem to be working out that way. No matter how many loose ends I tie up, my critics always manage to continue tripping over their own shoelaces. So I am back here again today to clarify.

To start with, you can see how I matched the Manhattan metric in my analysis, when I used this diagram.
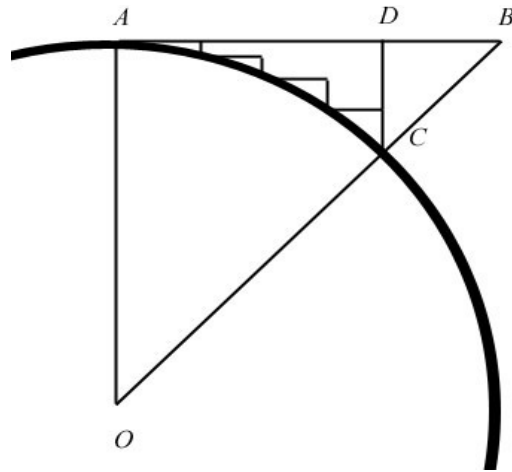
See how the little steps look like Hilbert's grid? Now, I hadn't *intended* to use any Manhattan method here: the grid I created was just an accidental match. I was trying to measure the curve with straight lines, not recreate any taxicab motion; but, as you see, the method is the same. My steps are like Hilbert's grids. The main difference is he wasn't trying to measure curves, and I am.

Even more important is what my method tells us about Euclidean geometry and the motion of real bodies. **Real bodies traveling curves do not travel the limit of the hypotenuses, they travel the limit of the orthogonal legs.** I state that outright in my original paper, but because I do not underline it there or put it under the title or shout it from the rooftops, almost everyone has missed it.

In the original paper, I say:

This means that the curve does not approach the hypotenuses of these steps, no matter how many there are. The hypotenuses are the chords, and they cannot be approached by the arc or tangent.

To put it still another way, the limit we are approaching when we increase the number of steps is not the sum of the hypotenuses. The hypotenuses are not the limit. The hypotenuses don't enter the correct math at all, neither the calculus nor the algebra nor the geometry. This is because—as I prove in an earlier paper—the tangent never approaches the chord as we go to the limit. The hypotenuse is the chord here, and the chord is *always* shorter than the tangent, even at the limit. Due to this, the chord is always shorter than the arc as well, even at the limit.

Modern physicists and other readers can't comprehend this, because they simply accept what Newton told them, and Newton told them that the tangent and the chord and the arc all went to equality at the limit. They don't, and I prove that in very simple fashion, with the simplest possible math and explanation. But rather than show where my proof or explanation fails, they shout that I haven't proved it to their satisfaction, in the terms they demand. As if I have to create a new proof for every person in the world, in their own preferred symbolisms, and tutor them on it personally until it penetrates their skulls.

Of course I don't. No scientist or mathematician was ever expected to do that. If they wish to comprehend what I have written, they should make some effort to do so. If they don't, they should read something else—something that confirms what they already think they know.

I say this with some heat, because I have witnessed my critics ignoring 9/10ths of what I say, purposely failing to take links, and then choosing to critique my glosses rather than my full papers. They huff and puff that my shorter gloss doesn't prove anything, claiming that I am just "handwaving" because the full argument isn't there. Of course the full argument isn't there: that is what a gloss is. It would be like reading only Einstein's abstract of Special Relativity, and saying that because he doesn't include an airtight proof in the abstract, he is a crank.

I also see many of my critics—one might say the bulk of them—claiming I don't understand calculus when it is clear *they* don't understand it. Their criticisms look incredibly lazy to me, which is why I haven't bothered to respond to them. What they commonly do is blast me because my analysis doesn't match the sound-bite analysis they were taught in school. They have developed some half-baked slipshod idea of calculus, most of it seemingly manufactured in their own minds from nothing, and because my historical analysis conflicts with that, I am a crackpot. It is clear from studying their criticisms that they have never bothered to study the historical progression of the calculus or any of the original proofs. They are only concerned that my analysis doesn't match what they were taught in high school, or doesn't match some video on youtube. But of course if they don't understand the original proofs of the calculus, it is very unlikely they will be able to follow my critiques of those proofs. To know whether a critique of A is right or wrong, you have to have a pretty good idea of what A is to start with, and none of my critics has that. Because they don't, they can only stomp around and make a lot of noise, hoping to keep others from looking at what I am saying.

Again, the proof that the tangent and chord don't approach equality is in my paper titled A Disproof of Newton's Fundamental Lemmae. You have to go there to get the proof, since I can't include everything I know in every paper I write. Given that proof, we find that the circle is NOT composed at the limit of chords or hypotenuses, as in Archimedes, Newton, or anyone after. Given motion and the time variable, the circle is composed at the limit of the orthogonal vectors. In other words, it is composed of the two shorter sides of the right triangle, not the long side. Which means that real objects in orbit travel a path that is represented not by the limit of the Euclidean metric, but by the limit of the Manhattan metric. And this means that in the kinematic circle, $\pi = 4$ and $C = 8r$.

What this means is that Hilbert's metric was never just a trick to get an answer when Euclidean geometry was failing to do so. As it turns out, the Manhattan metric is actually the correct metric in all orbital math, which makes it the correct math in both celestial mechanics and quantum mechanics. What is now called the Euclidean metric is in fact FALSE in most real kinematic situations, since the foundational math beneath it is compromised in many places. It is fudged. It is wrong. When it gets the right answer, it does so only by multiple offsetting pushes. I show that not only in my paper on Newton's lemmae, but in my papers on $a = v^2/r$, my papers on the calculus, my paper on $v = v_0 + at$, my paper on the Virial, and many others.

This also means that Hilbert could have discovered all this from his side if he had simply continued his analysis a bit further. Currently, the fact that $\pi = 4$ and $C = 8r$ in Manhattan geometry is seen as some sort of novelty, and no one has thought to take it as physically true and see where it leads them. It would lead them right to me. At Wikipedia, we are told that the circle looks like this in Taxicab geometry:
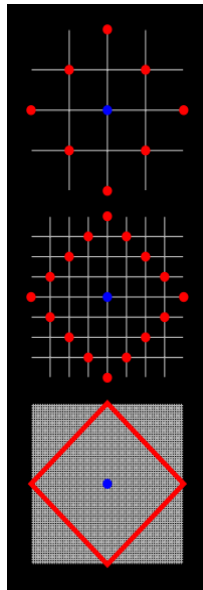
But that analysis and diagramming are misdirection. The circle is not defined—as they try to tell us— as "equal distance from an origin." The circle is defined as "equal *straightline* distance from an origin." Only four of their points in these diagrams are straight lines. The others are compound vectors in two dimensions. In truth, to create a circle in a Manhattan metric we would have to create it along the orbit, as I do, and it would look like this:
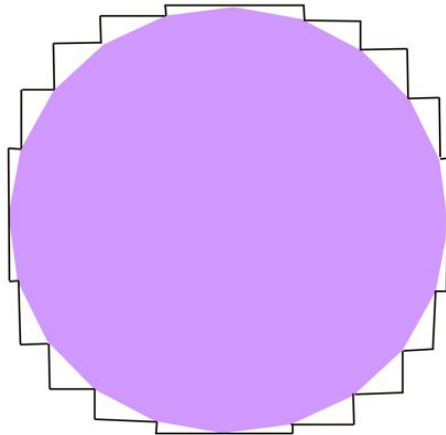
At the limit of 20 sides, the black line describes the Manhattan circle. As we take the number of sides to infinity, we approach the continuous circle we know and love. This means the circle was never the limit of inscribed polygons, as Archimedes proposed and is still thought and taught. It can't be because the polygon is composed of hypotenuses or chords, and the chords are never rectilinear vectors in our defined space here. The chords of a polygon will and must always be lines at some *angle* in this space, and angled lines are always compound lines, *by definition*. This is as true at the limit as it is at any other time. This space or metric is necessarily a representation of two dimensions, normally called x and y, and only lines in x or y are simple or non-compound distances. In other words, only vertical or horizontal lines are simple distances. Any slant is compound, and must thereby represent compound motions. Once Descartes gave us his graph to put our space in, we should have known this. But no one ever thought to put Archimedes' polygon into the Cartesian graph to analyze it this way, as I have.

Why does this matter? Again, it matters because velocity is *defined* in terms of simple distances, not compound distances.   In this representation, it doesn't matter if we call it Manhattan or Euclidean—in either case a slant must represent two vectors, not one. Well, if we have two vectors, we have two velocities, not one. Therefore, no velocity can be represented by a chord or hypotenuse. Velocities are simple vectors in *one* dimension only, therefore velocities can only be represented by vertical or horizontal lines. Any slant is already two velocities over the same time interval, and two velocities is already an acceleration. That is what an acceleration is: two velocities over the same interval, given time. So you see that any slant here already represents an acceleration, even without any curvature.

You will say, "Why do we care about velocities? I thought we were looking at circles or city blocks." No, motion is implicit in both my metric and Hilbert's. In the Manhattan metric, we are concerned with motion from one place to another. We are not concerned with a given static shape. And of course in my metric we are in an orbit or other kinematic situation. Kinematic means motion.

This affects all larger maths like QED and GR, because these maths represent forces, accelerations, curves, or energies, all of which depend on velocities. If velocity is not defined rigorously enough from the beginning, all these larger maths will implode. They *have* imploded, and this problem with velocity is a central reason why.

This is what I meant when I said in the previous update that "time adds a degree of freedom." In any kinematic situation, we have a time variable implied, existing underneath any other vector analysis. In the Hilbert metric or the Cartesian graph, we have a third vector existing unseen beneath x and y. This time vector turns every simple distance vector into a velocity and every pair of vectors into an acceleration.

But this time variable is a bit tricky, since it can mean two different things. We can apply time to x and y in two very different ways. We can either give x a time and y a time, or we can give them both the same time. In other words, we can let our taxicab travel x in time t_1, then let it travel y in time t_2. OR, we can let the taxicab travel x and y during the same time.   That is, during the same interval. In the first case, we get a Manhattan grid, as with Hilbert. In that case the sum of the two distances could either give us a Manhattan distance or a Euclidean distance, depending on whether we summed along the grid or along the slant. But in the second case, the summing of the distances would actually be an integral. We would integrate the motions into the same interval, *and would thereby obtain a curve*. Instead of a slant, we would have a curve and an acceleration. Working backward, we can then measure this curve by redrawing the x's and y's. The curve is measured and defined by the orthogonal vectors, not by the slants or hypotenuses. And this applies not only to circles, but to all curves. The length of any curve must be found by analyzing its orthogonal vectors in a defined space, not by analyzing chords or hypotenuses. This is because curves are a series of arcs, and arcs never converge upon chords, or the reverse.

Since the calculus ignores all this, it is flawed at the foundational level. The calculus is useful mainly as a measure of curves, and it normally measures them by summing slants or hypotenuses. But since the slants don't converge on the curve, this method must and does fail. This is one cause of the need for renormalization, but there are many others.

Most people think that contemporary math is either perfect or very close, but it is neither. In this problem of the circumference of the kinematic circle, current math is off by over 27%. Since the circle is mismeasured by that amount, we may assume that most curves are mismeasured by equal amounts.

Newton's mistake at the limit, based on Archimedes' mistake regarding polygons, has caused a 27% hole in so many equations that we may say that all of math and physics has a hole in it of that size. That is, more than a quarter of all math and physics is air, based on a single fundamental error. And I have catalogued dozens of fundamental errors of equal importance. I think you can now see how we end up with vacuum catastrophes, where we have compound errors of 120 orders of magnitude.



That said, I do understand why this paper on π has been so controversial. It not only conflicts with all we have been taught, it conflicts with basic intuition and with our own eyeballs. Even after I did the math and analysis, it was initially hard for me to accept. I am an artist and I rely on my eyes. I am extremely visual. It simply doesn't seem right that these two figures should have the same circumference:
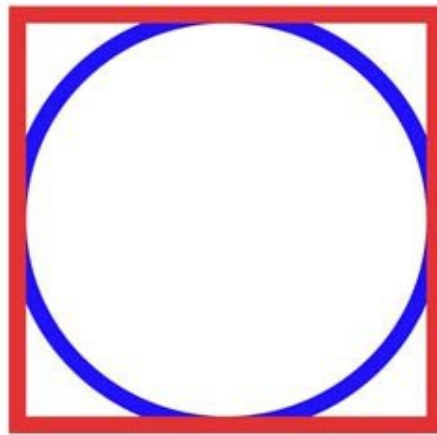

I will be told that any idiot can see they don't have the same circumference. The circle has to have a smaller circumference, since it encloses a smaller area. That is clear just from looking at the four spaces in the corners, which are outside the circle but inside the square. I used to think this way, and it is an intuitive way to think. However, it is mistaking circumference for area. Area is not a function of circumference, although you would think it would be. I can show you this very easily.
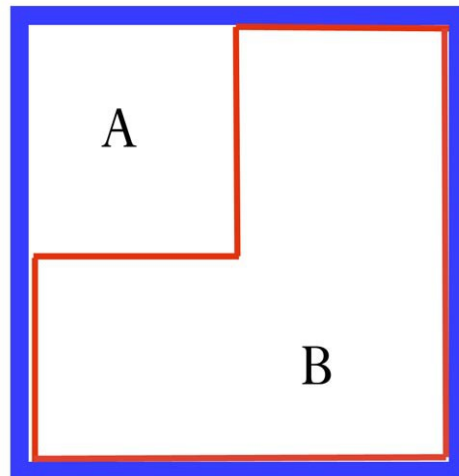
The red and blue lines have the same circumference, but very different areas. The area of red is ¾ the area of blue. And we can make the area of B go down very quickly, keeping the circumferences the same:
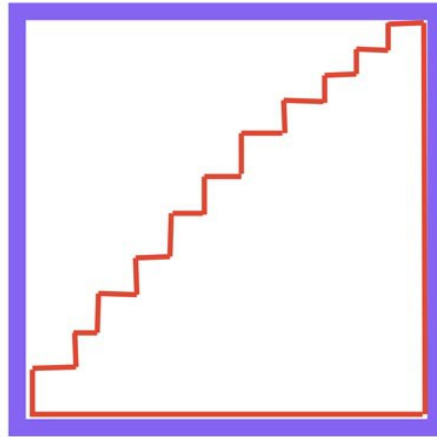
Red still has the same circumference as blue, but the area is approaching ½. This is precisely what is happening with the circle, although it isn't what we are taught.
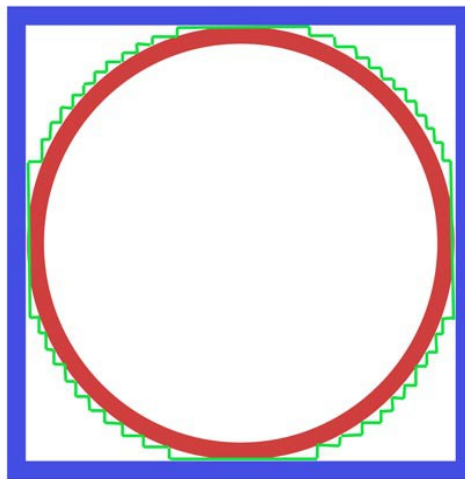
Study the green line. By the same intuition that told us that the red line must have a smaller circumference than the blue line, we should also expect the green line to have a smaller circumference than the blue. **But it doesn't!** I don't have to prove that or take anything to a limit to show it. Green has a much smaller area than blue, but exactly the same circumference.

It is pretty obvious that if we increase the steps in the green line, we will approach the red line. As the number of steps increases, the size of each step decreases, and by the logic of Newton, you could claim that "it would ultimately vanish." I don't like that wording and never have, since it isn't rigorously true. The steps never "vanish," they simply become negligible. "Without extreme magnification, they vanish": that would be a preferable wording.

You can prove this to yourself in photoshop. Click on your line tool and then click on the polygon image and choose a number of sides. How many sides do you think you would need before you couldn't tell the difference between a polygon and a circle? Photoshop will give you up to a hundred,

but you don't even need that many. About thirty will do it. At thirty sides, the difference between the polygon and circle has already vanished. But has it really? No, with magnification you can still see it. This is actually what "approaching a limit" is, in most situations. It is increasing sides or steps or manipulations to the point where they are negligible, without extreme magnification.

Problem is, we have two variant methods of approaching the circle, one with polygons and one with steps. You would think they would converge on one another, but they don't. They don't meet in the middle at all. By one method of going to a limit, we get π, by the other we get 4. And that is a huge difference.

Which is correct? Well, amazingly, we have never had to choose. Archimedes picked polygons early on, and history has followed his method. The step method never even came up, as far as most of us know. Even when Hilbert discovered or rediscovered the step method with his Manhattan metric, he never thought to apply it to the circle. Maybe he didn't wish to stir up the firestorm I have stirred up.

But, as I have shown, it is the step method that is correct. The red, blue and green lines above all have the same circumference. Very different areas, but equivalent circumferences.

And the reason the step method is correct while the polygon method is incorrect has to do with the way the field or metric or space is defined. As I said above, measuring the circumference as a limit of polygon sides requires we take slants or diagonals to a limit, and that cannot be done. Polygon sides are always diagonals in our space, and diagonals are always compound variables or compound vectors. You cannot take compound vectors to a limit in an ordered way, because, with the underlying time variable, they are actually curves or accelerations. A rigorous field solution requires we take simple variables or vectors to limits, and that can only be done with orthogonal or rectilinear vectors. This is precisely what our steps represent in the field. They are simple vectors, one that can be decomposed no further. An acceleration can always be decomposed into velocities, for instance, but a velocity cannot be further decomposed. A velocity uses a single distance, and you cannot get less than "single."

For further proof, I recommend you study this animation sent to me by a reader John McVay. He developed this from a similar animation he saw in an old 1986 *Mechanical Universe* segment on PBS. You can see this segment—originally produced by Caltech—at Annenberg Learner. Go to minute 11:15 of episode 9 to see the fuller animation. In both the Caltech and McVay animations, we see how the circle is produced straight from orthogonal vectors. In the Caltech animation, it is clear that no diagonal is ever produced: the motion is a simple addition of x and y. Neither x nor y ever move on a hypotenuse, therefore the combined motion cannot do so, either. This animation is just a speeded-up step method, and therefore *pi* would equal four here.

[I previously had a youtube link posted of this Mechanical Universe segment. Within a few weeks of my link, the youtube videos were removed by the poster. Here we see more obstruction of science by the mainstream. Rather than allow an alternate interpretation of data, they prefer to remove the data. Or, they post the data *only* as long as it only sells their own interpretation. If anyone discovers a better interpretation—one that undercuts the original propaganda—they have to remove the data. That is perfect anti-science. The scientific method would be either to counter the new interpretation, showing how it is wrong; or to allow both interpretations to stand, letting the readers decide which was stronger. But that isn't the method of the mainstream, and hasn't been for decades. They hide all negative data, and when any of their old "positive" data or demonstrations are questioned, they simply remove them

from sight as well. That is a pseudo-science grounded in authority, censorship, obstruction, and misdirection. These people prove my point with everything they do. **Addendum 2016**: they have now removed the video from Annenberg Learner as well. They should change the name of the site to Annenberg Anti-Science. I checked youtube to see if someone had reposted it there, but CalTech has actually deleted almost all its Mechanical Universe videos. As you see here, you can still subscribe to this old series, but once you get in, almost all the videos have been deleted. Man, have I got these people on the run!

Breaking News: a reader found a copy of the Mechanical Universe segment on DailyMotion.com. Also just posted again at youtube.com. Watch it while it is up. And anyone who has the capability should download it. They are sure to delete it again.

October 9, 2016: the youtube repost lasted 2 days after I posted my link. It has been removed due to a copyright claim by "Intelecom Intelligent Communications". Someone *really* doesn't want you to see that video. You may want to ask yourself why mainstream science is trying so hard to suppress one of *its own* educational videos. Before I came along, these videos were up on youtube for years.]

There are two other things worth mentioning while you have these animations before you. One, notice that the motions in x and y are both accelerations. Neither cursor is moving at a constant velocity. Each speeds up in the middle of each circuit and slows at both ends. This means that in this method, neither component is a velocity. In total, we have four velocities, two in x and two in y. Which makes the total circular motion a variable acceleration to the power 4. Two, the limit here is completely a matter of speed. The faster we move the cursors or the animation, the closer we come to any limit. Which means the limit is a matter of time t, not of x or y. By speeding up the animation, we are making the steps smaller, and taking the method closer to a smooth curve. But since we cannot speed up the animation infinitely, *the curve is never smooth*. It only looks smooth to us because we cannot see at those speeds. We can't even see at the speed of movie reels, which is relatively slow. The narrator at Caltech admits this, in a way, when he says that circular motion is a sort of trick of the eye:

Can the naked eye see reality perfectly? Not always. Sometimes perceptions of reality may be mere shadows of the real thing.

We see a curve where there is no curve, only a changing integration of x and y motions.

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The reason this solves many problems in QED and celestial mechanics is that most real bodies do not take shortest distances or diagonals. Since real bodies have multiple influences, they will have compound motions, the most common being the orbit. Remember, the orbit contains no diagonals. With Newton, the orbit was the compound of two rectilinear vectors, the centripetal motion and the tangential motion. That representation is a step representation, not a hypotenuse or diagonal representation. And Einstein did nothing to change that. He only imported time differentials into Newton's field. The same can be said for the quantum level, where the electron never travels diagonals. In its response to the nucleus, it is always in an orbit or other curve. Quantum mechanics is a step mechanics by its very nature, and in most ways that is admitted. But now we see it is not only a step mechanics in the respect of quantum levels, it is a step mechanics because it too must conform to the rectilinear nature of the math that describes it.

I will be told that modern mechanics is curved, not linear, but this is not strictly true. Even when the math is non-Euclidean, the curves still depend on an implied rectilinear field beneath the curved field. As I show in my paper on that, even the tensors have to be given a hook by the 4-vector field. Curvature would be meaningless without the idea of a straight line.

I now have physical proof that my step analysis is correct, since I have used it to correct many of the embedded problems of QED and celestial mechanics. Only by using 4 instead of π have I been able to do what I have done. The best example of that is my quantum spin equations, where I show how all quanta may be expressed as stacked spins. My spin equation couldn't match current data with π, but it did so easily with 4. Another example is my dissection of the fine structure constant. I show that by inserting 4 for π, many disclarities can be resolved, allowing us to transform directly between G, k, and the fine structure constant. I also corrected Bohr's equations in a similar way, including the known mismatch between the Bohr magneton and the magnetic moment of the electron.