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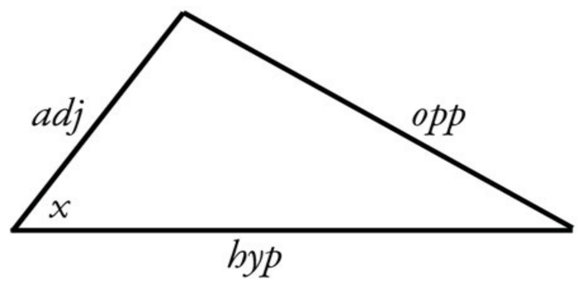
The Derivative of sin5*x*

how the chain rule works
and how it doesn't



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In a previous paper I stated that the current proof of the derivative of sin*5x* was pushed using the chain rule. Although I gave a brief account of how it fails there, it will help you to compare the pushed proof to a correct proof. This will clarify the many problems with the current and historical proof.

I showed in that paper that the chain rule wasn't really applicable to sin*5x* because that can't properly be fit to the chain equation

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

The relationship of sin*x* to 5*x* isn't that kind of relationship. But even if someone could convince you that it is, the current proof would still be a push for a still more fundamental reason. That reason concerns the fact that applying the chain rule to this term fails to monitor the correct relationships. I have shown that any time you are finding a derivative of sine, you also have to monitor cosine. By "monitor", I just mean you have to take into account the relationship of sine and cosine (and their changes), and explicitly include it in your mathematical manipulations. Any proof that fails to do that is a fudge.

Therefore, to solve this problem or any other problem concerning trig derivatives, it helps to start with an equation that already includes both sine and cosine. You can't manipulate both if both aren't already on the page. The mainstream sort of knows that, *sometimes*, since they have long solved for the derivative of sin*x* by starting with an equation that also includes cosine. With that in mind, in that previous paper I solved two basic trig derivatives by starting with the old defining equation

$\sin^2 x = 1 - \cos^2 x$

I will do the same here. As you may have already noticed, I have now dropped the convention of writing that with parentheses, since the parentheses are just used by the mainstream to fudge the equation.

Now, if we are seeking the derivative of sin*5x*, we can rewrite our first equation:

$\sin^2 5x = 1 - \cos^2 5x$

As before, we then differentiate the right side

$\Delta\sin^2 5x = -2\cos 5x$

Again, as my readers know, I simply use a delta to indicate change, rather than the current confusing notation, which is both varying and unwieldy. My notation—beyond being simpler and more direct—acts to remind us that we are actually representing the change of the functions on both sides; but on the left side we don't differentiate, simply representing the change with that delta. We do that to remind ourselves we are seeking the derivative of sine, not of cosine (and to remind ourselves that finding the derivative may take more than one step of differentiation).

So, we have monitored the change of cosine. Remember, the mainstream never does that in the current proof, which is one way we should have always known it was fudged. The mainstream manipulates sin*5x* twice, but never manipulates cosine in any way.

Now, since both sine and cosine were squared in the first equation, we need to differentiate $\sin^2$ as well. But as we do that, let's look ahead to see what else we need to do. Since we started by substituting *5x* for *x* in the first equation, we need to look at how that will affect our solution. We are allowed to do that to suit ourselves, but we can't do it and then ignore the consequences. For instance, if we originally had $x = 45$, say, then after we make the change, *5x* also has to equal *45*. Which makes *x* now equal *9*. If we do that, we see that sin*x* and sin5*x* aren't really scaling to one another. We need them scaled to one another, since we need to be able to relate them both to the same number line, which is of course based on the number 1. But by solving for 5x instead of x, we have thrown that scaling off. We won't be able to compare rates of change of the two trig functions directly unless we scale them to one another.

Obviously, that is very easy to do, since the scaler is just the number 5. But how do we legally work in that scaler? Well, there are various ways to do it, but one way is to work it into the rate of change math. Like so:

$\Delta\sin^2 5x = -2\cos 5x$

$\Delta\Delta\sin^2 5x/\Delta\sin 5x$ {with respect to $\Delta\sin x$} $= -2\cos 5x$

This time we don't put a delta in front of the right side, because we have already found the change over there. This second differentiation isn't really the reverse of the first differentiation, since the first differentiation was the change of $\cos^2$ relative to $\sin^2$. We don't need to reverse that, since we don't need to know the change of $\sin^2$ relative to $\cos^2$. Once we know the relationship in one direction, we know all we need to know in that regard. Rather, the "reverse" differentiation is the change of $\sin^2$ relative to sin (while importing the scaler). So we have to track deltas only on the left side.

I am writing out the expanded proof here, instead of the compressed proof I wrote for sin*x* in the previous paper. I do this to show that although you basically just differentiate both sides, you differentiate $\sin^2$ for a different reason than you differentiate $\cos^2$. You can see that this is very important in this case, because the denominator doesn't reduce to 1 here, as it did (or would have) with the proof of sin*x*. We use the denominator to import the scaler.

Concerning the actual operation: when you differentiate a term, you take away one of the deltas, since each delta is telling you that you *can* differentiate. After you actually differentiate, you don't need it anymore. So we are down to this:

$\Delta 2\sin 5x/5 = -2\cos 5x$

$\Delta\sin 5x = -5\cos 5x$

This also shows us where the *5* comes from in the final equation. The mainstream derives that *5* by differentiating *5x* as the second part of the chain rule, but you can now see how that was a fudge. You can't differentiate *5x* separately since it is not an interior part of the chain in that way. You can't separate it from its sin or cos signifier. And you don't *need* to, as I just showed. Strictly speaking, the number *5* doesn't even come from the rate of change math in the same way as the rest (which is yet another reason the chain rule is inapplicable here). It is imported into the equation after the fact, as a scaler. I have imported it as a rate of change to match it to the other notation, but you could just as easily import it without differentiating anything, as a raw scaler or constant.

Since sin and cosine change in opposite ways (see previous paper, footnote), we can drop the negative sign, obtaining this final equation:

$\Delta\sin 5x = 5\cos 5x$

Now to answer a couple of questions. Some of you may understand that my proof is still somewhat compressed, since we have to be careful how we treat those two deltas in the numerator on the left side. Some will say, "Why not just differentiate the numerator first in the second step, since then you would get $2\Delta\sin 5x/\Delta\sin 5x$? That would then reduce to 2." The reason you can't do that is because the second delta in the numerator is telling us that we are relating changes between the numerator and denominator. The deltas on top and bottom go together and have to be solved together. So you can't differentiate the numerator and then just stop. You have to differentiate them together, because we are monitoring how they change relative to one another. If you just differentiate one, you won't have discovered that, and your manipulation will be in vain. To indicate that, we probably need to tweak our notation just a bit. This might do it:

Instead of $\Delta\Delta\sin^2 5x/\Delta\sin 5x = -2\cos 5x$

Write it as $\Delta\Delta'\sin^2 5x/\Delta'\sin 5x = -2\cos 5x$

Various other simple notations might work as well.

The second question would go something like this: "In the second step, you divide only the left side by $\Delta\sin 5x$. How is that legal? Shouldn't you have to divide both sides by the same amount?" No, because, as I just showed you, that entire manipulation was done outside the given equality, as a scaler to a function outside the equation. We are scaling here to sin*x*, which never appears in the equation. And we are doing that so that we can compare the derivatives of sin*x* and sin*5x* directly after we solve.

You might ask, "Well, isn't that what the mainstream is doing with the chain rule?" In effect, yes. In theory, no. The mainstream instructs you to take the derivative of *5x* **because it is inside the parentheses.** But that isn't why you do it. The parentheses have nothing to do with it, and there is no interior in that way. You do it to scale the final equation to sin*x*, and they never tell you that.

Then I might be asked, "Well, even if it is a scaler, shouldn't you import it on both sides? Aren't you still breaking a rule?" No, because there is no way to import a scaler on both sides of an equation. You can only import it on one side or the other, and you import it where it logically goes. In this case, it logically goes on the sine side of the equation, since we are scaling to sin*x*.

Like the chain rule, I have two main manipulations here. So although you might call my solution a sort of chain rule, it is very different than the current chain rule regarding trig functions. Notice that in the current solution, no mathematical manipulations are done upon cosine. Cosine only enters as the derivative of sine. But in my proof, I make it clear that cosine has to be in the solution from the start. You will tell me that cosine is in the current proof implicitly, since to find the derivative of sin*x* the mainstream also manipulates cosine. But that manipulation is also a fudge, as I have shown elsewhere. The current method for finding the derivative of sin*x* is quite complex, and it relies on misusing infinitesimals or infinite series, in precisely the same way as in the fundamental proof of the infinite calculus. All you have to notice is how they are pushing the term *h* there, to understand how the entire proof is pushed. They push the proof of sin*x* just as the push the proof of the calculus itself.

We can see that by the way cosine enters my equation. In the current proof of the derivative of sin*5x*, cosine enters as the derivative of sine. In my proof, cosine enters as the differential of $\cos^2$. That is a big difference. We saw a similar thing in my previous paper, where I pulled apart the current proof of the derivative of $\sin^2 x$. In that case, we saw the number *2* entering the final equation from the wrong place, proving the current proof was fudged. Here we saw the number *5* entering from the wrong place, proving the same thing.

As I compile more and more of these specific pushes, it confirms more and more my basic contention that the foundations of the calculus have never been understood. If mainstream mathematicians had ever understood how these changes were working, or where they were coming from (see the table in my long paper), these pushes would have been impossible. But these pushes in the trig functions make me think that trigonometry has also been opaque in some ways at the foundational level up to the present time. If mathematicians really understood how sine and cosine were linked in an operational—and one might say *physical*—way, they would never have made the mistake of manipulating one without the other.

Modern mathematicians have proven themselves adept at manufacturing increasingly abstract and complex number systems and manipulating those systems in any way that suits them; but when numbers get near any physical situation, those same mathematicians have proven themselves very poor practitioners. Or, to say it another way, whenever math becomes applied math, the applications suddenly become slippery in the extreme. This can only be due to the fact that the mathematicians don't understand what their numbers actually *apply* to.

Trigonometry is the perfect proof of this, since trig is always an applied math. At its most abstract, it is applied to drawn figures, but at it most useful, it is applied to real objects and situations. But in either case, it is *applied*. It is never pure math. As I was composing the footnotes to my previous paper, it finally occurred to me that perhaps this knowledge of sine and cosine I was taking for granted—the true extent of their interdependence—was not actually known. Perhaps most mathematicians and physicists had never considered the impossibility of monitoring sine without cosine. Since my specialty is not history of science or math, I can't say that no one has known this; all I can say is that it seems curious that those who knew it would find trig derivatives the way they are currently found. Since they are currently found the same way they have always been found—roughly—it must mean that mainstream mathematicians and physicists haven't fully comprehended all the physical implications of either trigonometry or calculus, all the way back to Newton and Leibniz. That is not too surprising, considering the fact that applied math is still in its infancy, having been applied to complex engineering feats for only a few centuries. Still, the idea goes against all we are taught, since we are taught that math and physics are well nigh perfect and finished.

They aren't: not even close. When people ask me how I can go against all of history on questions like this, I laugh and tell them I am just playing the odds. When have human beings ever been right about anything? Human history is a history of ideas being overturned by better ideas, which are then overturned themselves. So the odds that any of our current ideas are complete or final are approaching zero. Anyone who says that current theory is wrong then has about a 1 in 1 probability of being right. Of course that doesn't make my corrections to current theory right, but since almost no one else is *offering* corrections, it automatically puts me ahead of the curve. If even one of my suggestions is correct, I will be ahead of those who offered nothing new.

As I have said before, that is why pronouncements like the Copenhagen Interpretation should have been laughed out of the lab, without further consideration. Those guys were promoting probability math without being able to do it: the probability that math and physics of the 1920's was the best we would ever do was zero. How shortsighted for those in 1926 to think that *anything* was finished in 1926. What was so special about 1926, other than that they were alive then and making up rules?

As I have said before, “question authority” isn't just a bumper sticker. It should be the first principle of all science.