return to updates

More Problems with the Friedmann Metric

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The Friedmann (or FLRW or Robertson-Walker) metric is a centerpiece of applied Einstein field equations, being used for many things in astrophysics, including—most famously—calculating the expansion of the universe. It is the starting point of all inflationary models, including the current ones.

The generic metric—or starting equation—of the FLRW metric is written like this

$$-c^2d\tau^2 = -c^2dt^2 + a(t)^2d\Sigma^2$$

I will show that equation is a complete hash, compromising everything after it.

To simplify my analysis and make it comprehensible to the majority of my readers, I will first simplify that equation. The variable c is the speed of light, of course. The variable τ is another way to write t', which is the other coordinate system in the Relativity transform. Relativity transforms from t' to t. To simplify, we can take the square root of both sides, getting rid of all the squares. We can multiply through by -1. We can write it as algebra rather than calculus. And we can also ignore the last term in my initial analysis, since we will be looking first at the other terms only. That brings us down to this:

$$ct' = ct - A$$

That is clear enough, I think. Now, we want to start by analyzing those first two terms, ct and ct'. If you have read my Relativity papers or Einstein's, you know that both Einstein and Lorentz started with these two equations:

$$x = ct$$
$$x' = ct'$$

So, the generic metric of the Friedmann equations can be boiled down to

$$x' = x - A$$

Couldn't be much simpler than that. But back to the previous equations x = ct and x' = ct'. Those are called the light equations of Special Relativity. After the equation x' = x – vt, those two equations are the first equations of Special Relativity. They stand for the distance light goes in the respective coordinate systems S and S'. Those are the axiomatic equations of Einstein and Lorentz, since they start with those equations but do not prove them. As I have shown, axiomatic equations tend to be less scrutinized than the equations that follow them, for reasons that don't make much sense. When physicists or mathematicians plop down a proof, most other physicists and mathematicians start analyzing the math *after* the axioms. They leave the axiomatic equation or equations alone, since it is known to be an assumption. It has the "let" in front of it, so they just let it be. That is what has happened historically to these equations. We have had millions of pages of arguments about the proofs, but almost no pages of arguments about the assumptions. In other words, no one has looked at these two little light equations, to see if they make sense. They don't.

To begin with, they contradict Einstein's postulate 2: "light moves... with the same velocity... whether the light is emitted by a body at rest or in motion." That is, c is always c, no matter what co-ordinate system you measure from. **The constancy of light**. And yet here, we find light needing two equations, two times, and yielding two different distances, x and x'. Einstein tells us Relativity applies to everything except light, and here he is applying a transform to light! His first equations contradict his second postulate.

To say it in a different way, light can't travel in S or S'. For light, there are no separate coordinate systems. Light simply travels. It is the motion of light that *creates* S and S', so light's own motion can't be assigned to either one. You can't assign t or t' to light. Since all coordinate systems measure light to go c, you can't find any time differentials for light itself. Since there is no t or t' for light, there is also no x or x' for light.

Relativity can't apply to light itself, since Relativity is *caused* by the constancy of light. If we apply the transforms to light, we have created a circular argument. But it is even worse than that. Say you don't understand any of what I just said. Well, if we disregard all I just said and go ahead and let these simple transforms apply to light, we still have the constancy of light postulate, which means that, at the least, c=c. Which means we can combine the two light equations like this

$x = ct$
$x' = ct'$
$c = c$
$x/t = x'/t'$

Therefore, x and t are in direct proportion, as you see. As x gets larger, t must get larger also. But that isn't what Relativity purports to find or prove. As we know, in Relativity, x and t are in *inverse* proportion. In Relativity, we have time dilation and length contraction. Dilation is a lengthening of the interval and contraction is a compression of the interval. In the current equations of Relativity, we have

$xt = x't'$

That is an inverse relationship. We can see this from the current transforms underlying the tensor calculus:

$L = L_0/\gamma$
$t = t_0\gamma$

Length and time are in inverse proportion. In one equation, the term gamma (γ) is in the numerator and in the other it is in the denominator. Inverse.

What this means is that the light equations are wrong. The equations $x = ct$ and $x' = ct'$ cannot be used together in a logical fashion. I have shown that Relativity can be corrected, and that logical transforms can be found that aren't that different than the current ones; but these light equations have to be thrown out. The proof has to be rewritten from the ground up.

Of course this destroys the generic metric above, since those ct terms rely on Einstein's axioms. Friedmann, like Einstein and Lorentz, assumes you can substitute ct for x and ct' for x', but you can't. *Light does not travel like that.*

Now let's look at the third term in the generic metric, which is $a(t)^2 d\Sigma^2$. At its simplest, $d\Sigma^2$ can be written as

$$d\Sigma^2 = dx^2 + dy^2 + dz^2$$

Since x, y, and z are lengths, Σ must also be a length.

Wikipedia tells us

It [dΣ] is normally written as a function of three spatial coordinates, but there are several conventions for doing so, detailed below. dΣ does not depend on t — all of the time dependence is in the function a(t), known as the "scale factor".

This is curious, since we find a(t) multiplied here by dΣ. We are told a(t) contains the time variable in some way, but it is being multiplied to x,y,z. Does that make any sense? To answer that, let us look at the Minkowski 4-vector simplification, which is written like this:

$$c^2t^2 - x^2 - y^2 - z^2 = 1$$

There, the time term is *added* to the equation, not multiplied. If time is a “fourth vector”, why would you multiply it into the mix? We are told that time can be treated like the other three vectors, giving us “four dimensional space.” But in this space, we don't multiply dimensions together. Distance in this metric isn't x times y times z, so why would we multiply by t, which is supposed to be dimensionally equivalent to the other three?

More problems are encountered when we are told that the scale factor a(t) should have dimensions of length or be dimensionless. It can't have dimensions of length, because if it did, then the term $a(t)^2 d\Sigma^2$ would have dimensions of L^4. That can't work, because the other two terms in the equation $c^2d\tau^2$ and c^2dt^2 have dimensions of L^2. You can't *add* L^4 to L^2 and get L^2. But it can't be dimensionless either, since something that is dimensionless can't have time dependence. To be time dependent, it must include some function of time. That is what (t) means. A dimensionless quantity cannot be time dependent, by definition. You wouldn't think I would have to be here telling these guys that.

In fact, as they admit at Wikipedia,

$$a(t) = d(t)/d_0$$

Since d_0 is "the distance at reference time t_0", and d(t) is the proper distance, a(t) is still dimensionless. They try to divert you from that realization with the (t) tag, but it is true regardless. Once you divide a distance by a distance, your time function evaporates. The variable a *can't* have any time dependence there.

No matter how you slice it, the equation fails to make any sense. For the dimensions to work, a(t) needs to be dimensionless. But a(t) can't be dimensionless and carry any time dependence. In the current uses of the equation, the term a(t) isn't just time dependent, it actually carries the time variable. It tells us *how fast* things are expanding, so it includes time to some power. Which means the term is some sort of acceleration. But it can't be according to the form of the equation. It doesn't add up. You can't add an acceleration term and still get a distance.

On the other hand, if a(t) is dimensionless, then the generic metric has no time variable nor any time dependence. At that point, the equation contains only constant velocities like c, which can't even be differentiated. Not only does the equation contain no forces or fields, it doesn't even contain any curves. Please notice that when the generic metric is expanded and solved, it is expanded through the term a(t). But if a(t) is a dimensionless entity, it can't be expanded that way.

Since the generic metric boils down to the form x' = x – A, A must be a distance. Since A is composed of a term Σ that equals $\sqrt{(x^2 + y^2 + z^2)}$, Σ must also be a distance. Therefore, a(t) *should be* dimensionless. But it can't be and carry the time dependence. If we want time dependence in the metric, we have to get it in there in a logical way. We would do that by making all the lengths into motions, which would make them velocities. Which means we need to divide through by time t, making *all* the motions velocities. All three terms would be velocities.

However, let us back up, and ask if the equation makes any sense to start with. If $\Sigma = \sqrt{(x^2 + y^2 + z^2)}$, then Σ is some hypotenuse in either S or S'. That is, it is a distance in either S or S'. We have no other co-ordinate systems in this equation, so there is no other choice. Therefore, Σ must be either some x or some x'. In that case, the equation boils down to the form

$$x' = x - ax$$

Since x' equals ct', we are being told that the distance light goes in S' is equal to the distance light goes in S, minus some other distance in S. Again, that must be nonsense, for many reasons. One, light has to go the *same* distance in S as it goes in S', otherwise the constancy of light is meaningless. Two, because the x in the term ax is just a restatement of the term x, we are trying to subtract something from itself. In other words, we can write the second term x in the equation as either x = ct, or we can write it as

$$x_t = \sqrt{(x^2 + y^2 + z^2)}$$

In that case, we take x as some sort of x-total, which we can then write as a compound of the three orthogonal dimensions. If we re-expanded that equation, it would be

$$x' = \sqrt{(x^2 + y^2 + z^2)} - a\sqrt{(x^2 + y^2 + z^2)}$$

As I hope you see, none of this makes any sense at all.

It makes no sense because it is derived from the three initial equations of Special Relativity:

$x' = x - vt$
$x = ct$
$x' = ct'$

Look very closely at that first equation. It is surpassingly curious that the generic metric of the Friedmann equations mirrors the form of the first equation of *Special* Relativity. You should ask, "Shouldn't the Friedmann equations start from General Relativity, not Special Relativity?" Why would the generic metric mirror the SR metric or axiom? Watch this:

$x' = x - vt$
$ct' = ct - vt$
$c^2t'^2 = c^2t^2 - 2cvt^2 + v^2t^2$
$-c^2t'^2 = -c^2t^2 + 2cvt^2 - v^2t^2$
$-c^2d\tau^2 = -c^2dt^2 + (2cvdt^2 - v^2dt^2)$

The generic metric was

$-c^2d\tau^2 = -c^2dt^2 + a(t)^2d\Sigma^2$

As you see, Friedmann must have gotten the generic metric from the first equations of SR. That is odd, since SR doesn't even include gravity, much less E/M. It is nothing but time differentials. So, does that mean that

$2cvdt^2 - v^2dt^2 = a(t)^2d\Sigma^2$

No that is just more gobbledygook. The generic metric is a generic hash, in each part and in all parts.

We are told,

Einstein's field equations are not used in deriving the general form for the metric: it follows from the geometric properties of homogeneity and isotropy.

No, as we have just seen, it is derived from the very first equation of Special Relativity. Which means the above statement is false. Special Relativity underlies General Relativity, so Einstein's equations *are* used to derive the general or generic form of the metric. But only the time differentials are included.

This makes the generic metric the oddest of beasts. It doesn't come out of the *final* equations of SR, which would be bad enough (since they are compromised). It actually comes out of the *very first* equation of SR, which has never been justified from the beginning. This equation—which I have shown has absolutely no physical pedigree—which was just dreamed up by Woldemar Voigt back in 1887—was somehow and for some reason used by Friedmann to create this pathetic generic metric. In his proofs, Einstein started with the equation $x' = x - vt$, and then pushed it and pulled it to make it conform to data. But Friedmann apparently took it without any of Einstein's subsequent pushes, in its naked form so to speak. He remodeled the last term vt by some magic into a term in the form a(t)x, and renamed the equation the generic metric. All of his subsequent pushes enter through that trapdoor a(t).

In *solving* the Friedmann equations—determining the time evolution—we are told we have to use Einstein's field equations (GR). We need them to pull in the energy-momentum tensor, and thereby import the field forces into the equation. The term a(t) is the conduit for importing all that. This might have worked if the generic metric had been correct to start, and if Einstein's field equations were correct. But since I just showed how the generic metric fails, importing the force fields (or momenta) cannot save them. This is doubly true since Einstein's field equations are compromised by the same mistakes at the axiomatic level. Which means the Friedmann metric is compromised twice by the same mistakes. This problem with x = ct enters with the generic metric, and then re-enters with the energy-momentum tensor and the rest. Since I have shown in previous papers the many mistakes in GR, I have already done enough to falsify the Friedmann equations. Like everything else, they need to be rewritten from the ground up.

In conclusion, we can use all I have found to attack Guth's theory of inflation, and all other current theories that rely on the Friedmann metric—which would be all of them. Not only have I just proved that all their math and theory is completely unsupported, I have proved that they never understood the field equations to start with. Any physicist who had a basic understanding of kinematics, mechanics, or the forms of the equations of motion, would have seen at a glance that these equations of Friedmann and the rest were just fudge. I have never taken one day of tensor calculus or graduate-level physics, but I could spot the problems here within a matter of minutes. Why have no top-level physicists or mathematicians been able to do this? I will tell you why. They can't do so because they have been indoctrinated, not educated. They are told to memorize this stuff and not to question it. It never occurs to them to analyze any of these equations. Their job isn't to analyze anything that came before them. It is to extend all they have been taught into newer and larger data holes. The bigger and more complex the theories they can manufacture from all these fudges, the more famous they will be.